MAE 343. Intermediate Mechanics

Section 3-4: Stress III : 2-plane Bending Circular Section

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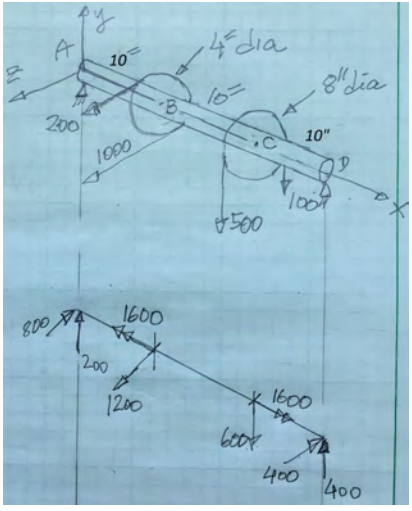
Review

$$\sigma_n = N/A$$

$$\sigma_b = \frac{-y\, M_z}{I_z}$$

$$\tau = \frac{T\, r_0}{J}$$

Example 3-9. Find max σ, min σ, and max τ



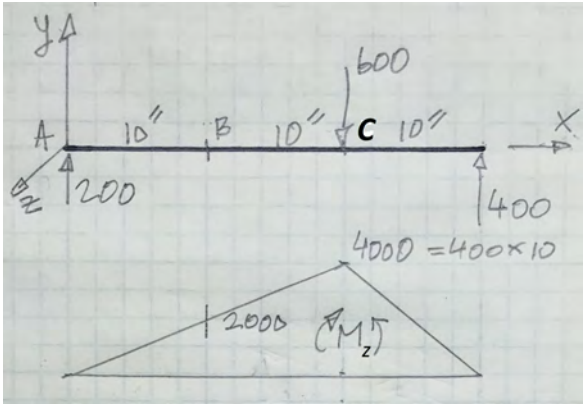
$$F_B = 1000 + 200 = 1200 \text{ lbf}$$

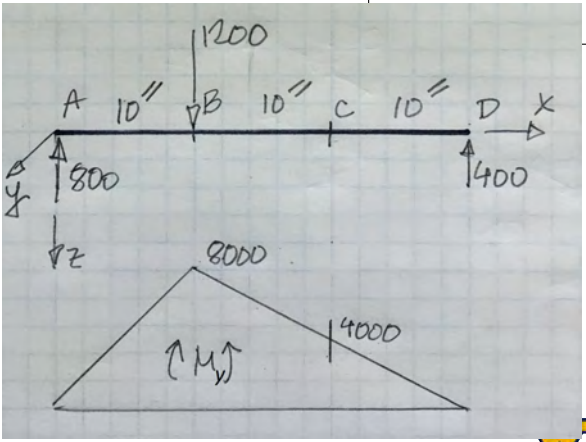
$$F_C = 500 + 100 = 600 \text{ lbf}$$

$$T = (500 - 100)\,4 = (1000 - 200)\,2 = 1600 \text{ lbf in}$$

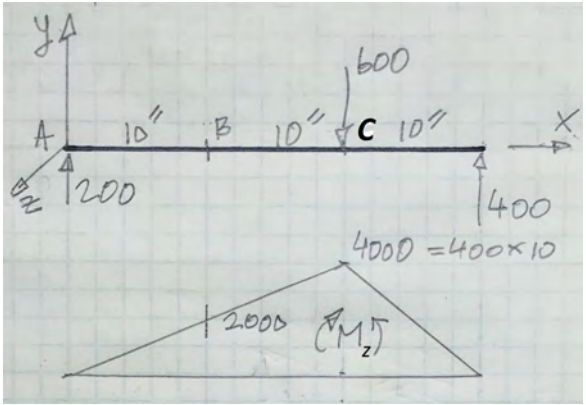
$$\tau = \frac{16T}{\pi D^3} = \frac{16 \times 1600}{\pi 1.5^3} = 2414 \text{ psi}$$

2 sets of diagrams





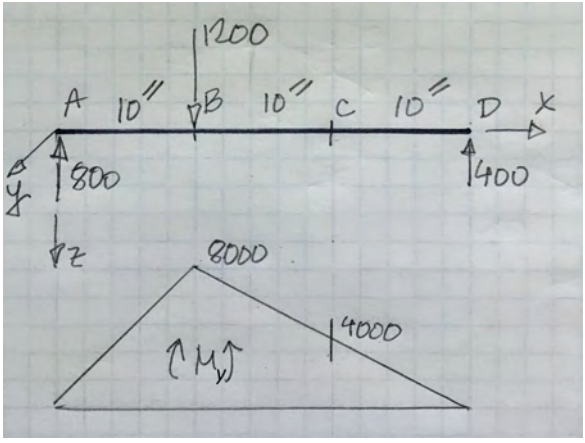
Stress components @ point C



@ C: $M_C = \sqrt{M_y^2 + M_z^2}$

$M_C = \sqrt{4000^2 + 4000^2} = 5657$ lbf.in

Resultant moment @ point B



@ B: $M_B = \sqrt{M_y^2 + M_z^2}$

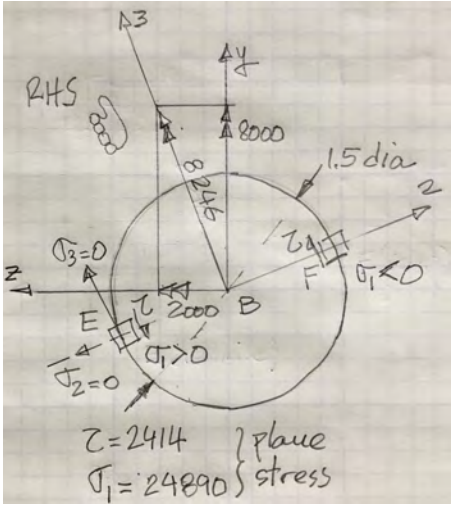
$$M_B = \sqrt{8000^2 + 2000^2} = 8246 \text{ lbf.in}$$

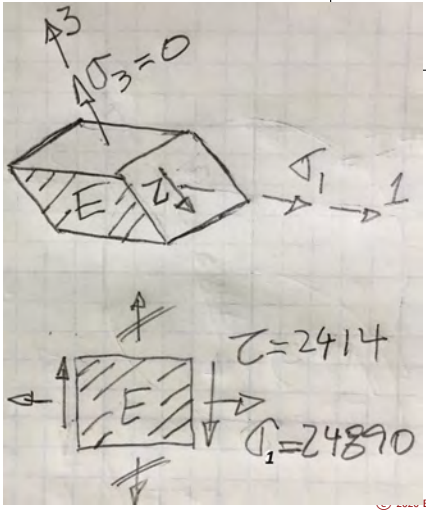
$$M = \max(M_B, M_C)$$

$$M = \max(8246, 5657) = 8246 \text{ lbf.in}$$

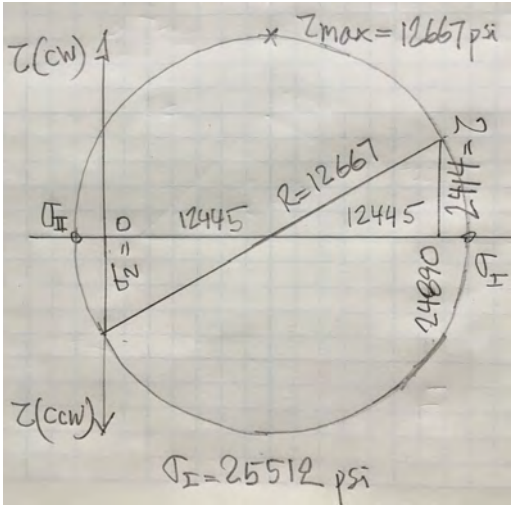
$$\sigma = \frac{32M}{\pi D^3} = \frac{32 \times 8246}{\pi 1.5^3} = 24890 \text{ psi}$$

Resultant moment @ point B





Maximum stresses at point B



$$\sigma_x = 24890 \text{ psi} \quad ; \quad \tau = 2414 \text{ psi}$$

$$\max \tau = R = \sqrt{(24890/2)^2 + 2414^2} = 12667 \text{ psi}$$

$$\max \sigma = \sigma_A = 12445 + 12667 = 25512 \text{ psi}$$

$$\min \sigma = \sigma_B = 12445 - 12667 = -222 \text{ psi}$$

Safety factor η

$$\tau_{max} = 12667 \text{ psi}, \quad \sigma_A = 25512 \text{ psi}, \quad \sigma_B = -222 \text{ psi}$$

Ductile metal, Tresca failure, $S_y = 36000$ psi

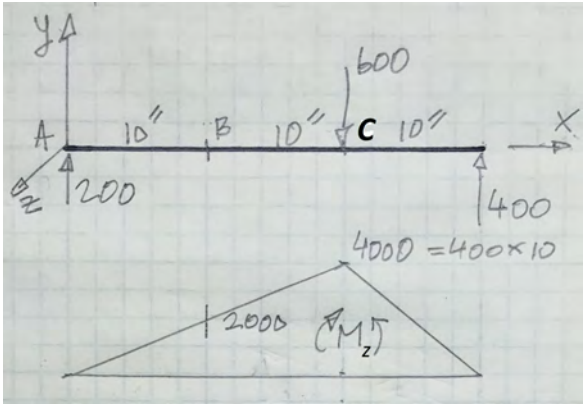
$$\eta = \frac{strength}{stress} = \frac{S_y/2}{\tau_{max}} = \frac{18000}{12667} = 1.42$$

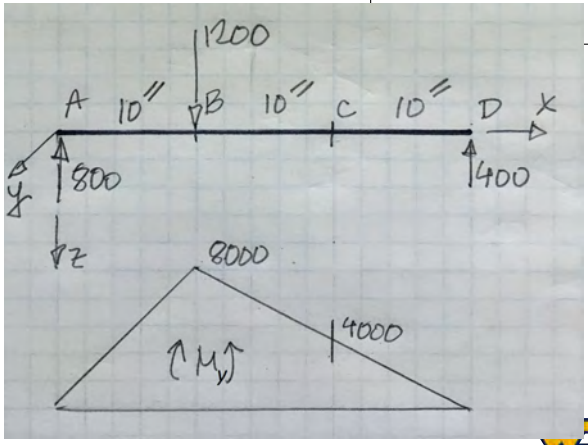
Ceramic, Brittle Mohr-Coulomb failure, case II (tension-compression), $S_{ut} = 36000$ psi, $S_{uc} = 72000$ psi

$$\frac{1}{\eta} = \frac{\sigma_A}{S_{ut}} - \frac{\sigma_B}{S_{uc}} = \frac{25512}{36000} - \frac{-222}{72000} = 0.712$$

$$\eta = 1.40$$

2 sets of diagrams





Summary

- Loads on 2 planes? no problem. Just do 2 FBDs, and 2 sets of V/M diagrams
- Calculate stress at all points along x that have a maximum in either Moment diagram (see "2 sets of diagrams")
- For a circular shaft, find the vector resultant M, and use that in $\sigma = -yM/I$
- Make a sketch to find the tensile and compression sides
- use Mohr's circle to find the $\max \tau$ and $\max \sigma$
- calculate the safety factor (chapter 5)

Next lecture

- 2-plane bending, non-circular sections
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