

MAE 343. Intermediate Mechanics

Chapter 3: Statics Internal Forces in Beams

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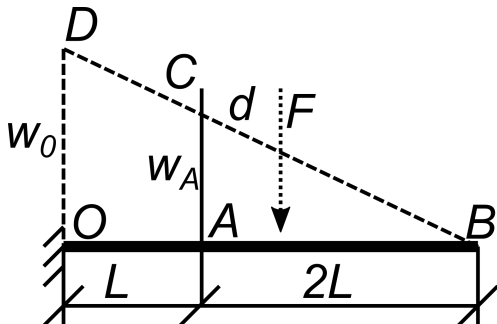
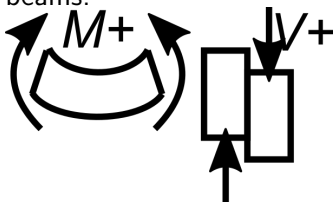
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Example 1

A cantilever beam is loaded with a triangular-distributed load $w(x) = w_0(1 - \frac{x}{3L})$. Calculate the internal forces at $x = L$ using the sign convention for beams as shown in the figure.

$w_0 = 240 \text{ N/m}$, $L = 2.8 \text{ m}$

Sign convention for beams.



Example 1, cont'd.

$$OD = w_0 = 240 \text{ N/m}, L = 2.8 \text{ m}$$

- Shear force at A = load on right of A

$$AC = w_A = 2/3 w_0 = 160 \text{ N/m}$$

F = Area triangle

$$= 2Lw_A/2 = 2 \times 2.8 \times 160/2 = 448 \text{ N}$$

$$V = F = 448 \text{ N (+)}$$

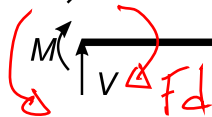
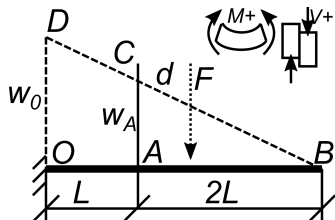
- Bending moment at A

Distance from A to CG of triangle

$$d = 2L/3 = 2 \times 2.8/3 = 1.867 \text{ m}$$

$$M = -Fd = -448 \times 1.867 = -836.3 \text{ N.m}$$

- Axial force $N = 0$



$$M = -Fd$$

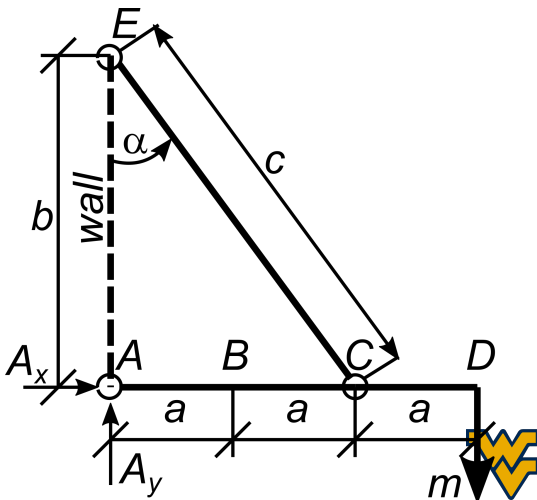

Example 2

The cantilever beam AD is loaded with hanging mass $m = 500$ kg at D, pin-supported at A, and held by strut CE. Calculate the internal forces at B.

Dimensions $a = 0.5$ m,
 $b = 2.0$ m.

Points A, C, and E are pinned.

The strut EC is pinned at both ends.



Example 2, cont'd.

Calculate the internal forces at B.

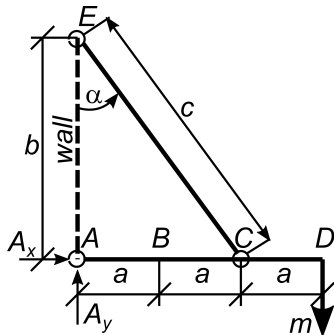
Mass $m = 500$ kg. $a = 0.5$ m, $b = 2.0$ m.

$$W = 9.81 \times 500 = 4905 \text{ N}$$

$$c = \sqrt{4a^2 + b^2} = 2.236 \text{ m}$$

$$\cos \alpha = b/c = 2.0/2.236 = 0.8945$$

$$\sin \alpha = 2a/c = 2 \times 0.5/2.236 = 0.4472$$



Example 2, FBD

$$\sum M_A = 0: 3aW - 2aF_{CE} \cos\alpha = 0$$

$$F_{CE} = 8222 \text{ N}$$

$$A_x = F_{CE} \sin\alpha = 3678 \text{ N}$$

$$A_y = -F_{CE} \cos\alpha + W = -2452 \text{ N}$$

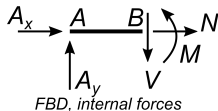
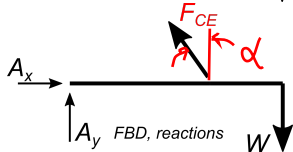
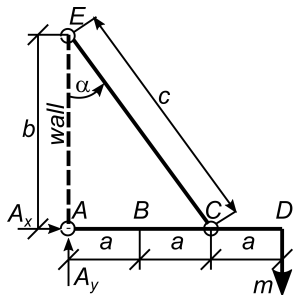
Internal forces at B:

$$N = -A_x = -3678 \text{ N}$$

$$V = A_y = -2452 \text{ N}$$

$$M = A_y a = -2452 \times 0.5 = -1226 \text{ N m}$$

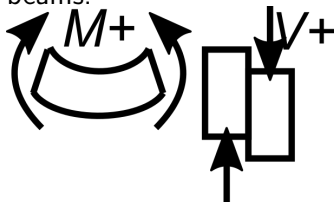
w/sign convention



Review Sign Convention

- ▶ The sign convention is such that M is positive for a simply supported beam under its own weight.
- ▶ The sign of the shear is such that $V = dM/dx$
- ▶ You should try and draw the shear and bending moment diagrams for a simply supported beam under its own weight, to convince yourself of these facts.
- ▶ Axial force N is positive in tension, negative in compression.

Sign convention for beams.



Conclusions

- ▶ Homework is on WebWork
- ▶ Next lecture: Shear and Bending-moment Diagrams
- ▶ THANK YOU

